Work in Progress - Structure Editing of Handwritten Mathematics

Alexandra Mendes
University of Nottingham, afm@cs.nott.ac.uk

Abstract - **This project aims to develop a pen-based software tool that will assist in the process of doing mathematics by providing structured manipulation of handwritten mathematical expressions. The tool will be used to support the teaching of the dynamics of problem solving in a way that combines the advantages of the traditional blackboard style of teaching with the flexibility and accuracy of computer software. It will provide not only a simpler way to input mathematics - by allowing the recognition of handwritten mathematics - but also enhance students' understanding of the calculational techniques and facilitate the process of doing mathematics - by providing structure editing. Some of the most important features of this tool are the accurate selection and copy of expressions, the automatic application of algebraic rules and the use of gestures to apply them, and also the combined writing of mathematics and text. These features will have a major impact on writing, doing, and presenting mathematics. This project includes the required technical developments and also the application and testing of the tool in concrete situations, namely in mathematics and computing science courses.**

Index Terms - Algorithmic Problem Solving, Calculational Method, Computer Science Education, Pen-based Software.

MOTIVATION

Writing mathematical documents and doing mathematics on a computer is a difficult task. The main problem is the input devices that are usually used: the keyboard and mouse. Due to this difficulty, people are more inclined to use the traditional pen and paper, which is less flexible than a digital document, but much easier to use. With the advent of pen-based technologies, like the Tablet PC, writing mathematical documents easily in a digital format seems more feasible. However, if we use a Tablet PC just to write documents as we can do on a piece of paper, then there is nothing much to gain from it, apart from the digital representation. The ideal would be to combine the simplicity of writing mathematics naturally and easily using a pen with the advances and flexibility that a computer has to offer. This is exactly our goal.

This tool is being created to support the area of mathematics of program construction [1]. In this area, doing mathematics involves a substantial number of systematic algebraic calculations. Thus, the main formulae that this tool will support are the ones used in this kind of calculation and our attention will be mainly focused on their systematic nature.

A twin project on principles and foundations of algorithmic problem solving will provide some of the "mental" tools which this project aims at animating in the form of innovative tools for school teaching support and industrial application. One of the main goals of these two projects is to improve the way mathematics is taught.

GOAL

Our goal is to create an application oriented to the mathematics used in algorithmic problem solving that is capable of recognizing and editing mathematics and text written on a Tablet PC's screen. This application will assist in teaching the dynamics of algorithmic problem solving and also assist in the process of doing mathematics. This tool innovates because it will allow straightforward and reliable manipulation of handwritten mathematical expressions. In the currently available applications, modifying handwritten documents containing mathematics is mostly unreliable. The problem is that the structure of the formulae is not represented correctly in these systems; for instance, copying parts of expressions can be very difficult or even impossible. Doing mathematics is a systematic process in which many steps of the calculations only modify sub-expressions. So, if the copy-and-paste of expressions is reliable and easy to use, then the simpler it is to do calculations. Also, the modifications performed in the sub-expressions are obtained by applying certain known rules. Applying those rules is (most of the time) nothing more than manipulating the structure of the expressions. In a system that represents the structure of the formulae correctly, it is possible to provide reliable selection and copy, and also reliable manipulation. With this application, the user will be able to apply several known rules automatically to handwritten expressions. By automatic we do not mean making calculations automatically; we mean that the tool will modify the structure of an expression according to the rule that the user wants to apply in a certain step. The user is responsible for guiding the calculation by deciding when and where rules should be applied. This, combined with the accurate selection/copy of structure, will help the user writing and performing calculations.

An example of a rule that is commonly used in algebraic calculations is the distributivity of disjunction (\/) over conjunction (/\):

$$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$$

In this tool, we will have an option or a gesture that applies this rule, transforming the right-hand into the left-hand side. To illustrate this, consider the handwritten formula shown in Figure 1.

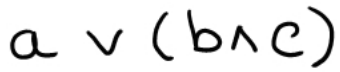

FIGURE 1
INITIAL FORMULA

A possible gesture to apply the distributivity rule is to select the □ symbol, drag it and drop it over the □, as depicted in Figure 2.

FIGURE 2
GESTURE FOR APPLYING THE DISTRIBUTIVITY RULE

The final result is then obtained automatically; Figure 3 shows the resulting handwritten formula.

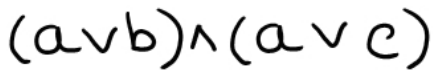

FIGURE 3
FINAL FORMULA

In our view, the use of gestures is a very clear and effective way of doing, presenting and teaching mathematics. Similarly, many other rules will be available in their general form with associated gestures that will apply them automatically.

In essence, our objective is to develop a system akin the Math∫pad system [3], which is a structure editor for mathematical documents that has proved very convenient for writing research articles across a wide spectrum of computing science. The main difference is that, instead of using keyboard/mouse input and manipulation, our tool will use handwritten input and pen gestures to manipulate a document and give the user the freedom to write mathematics without having to think in advance about the structure to be entered.

CURRENT STATUS

This project, which is being developed using the Microsoft Tablet PC API [4], is still in an early stage of development. However, it is already capable of recognizing arithmetic expressions and creating accurately their internal structure. Thus, it is possible to copy and manipulate structure in an easy and reliable way. Shortly, we aim to support several types of expressions used in algebraic calculations. In particular, we aim to support a substantial amount of logical expressions and quantifiers.

EVALUATION PLAN

This system is planned to be evaluated in real-life teaching situations by being used in mathematics and computing science courses. Such experiments will be carried out by independent teachers both at the secondary school and university level. This will be achieved with the collaboration of the University of Minho in Portugal. On the 29th of March 2008, a workshop will be held at the University of Minho where secondary school Mathematics teachers will be introduced to the way we teach mathematics and will also have a preview of this system. This will be useful to assess their reaction to our tool, and also to receive feedback on our work. This workshop is the first of a series of events where teachers and students will be exposed to our method and to this tool. Apart from these events, this software will be actively used in lectures at the University of Nottingham and University of Minho.

CONCLUSIONS

The benefits of using this tool for teaching algorithmic problem solving can be enormous. It will be possible to demonstrate the dynamics of problem-solving in a manner that improves on blackboard-style teaching by exploiting the reliability of computer software in copying and manipulating structured information. Also, students can benefit from this tool by using it to perform their own calculations.

Our main goal is to support the teaching of algorithmic problem solving but, as already mentioned, this tool will also assist in doing mathematics and help the writing of mathematical documents.

Although in an early stage of development, we are confident that this tool will be very useful for researchers and that it will have a great impact in teaching/learning mathematics.

ACKNOWLEDGMENT

The author of this paper would like to thank Fundação para a Ciência e a Tecnologia (Portugal) for the PhD grant SFRH/BD/29553/2006 and Maplesoft for allowing the use of their handwritten character recognizer. She also thanks to her supervisor Roland Backhouse and João Ferreira for their comments on earlier drafts of this report.

REFERENCES

[1] Backhouse, R, C, "Program Construction", *John Wiley and Sons, Inc.*, 2003.

[2] Dijkstra, E, W , "Guarded commands, non-determinacy and formal derivation of programs", *Commun. ACM*, 18, 8, 1975, pages 453-457.

[3] Backhouse, R, C, and Verhoeven, R, "Math∫pad: A system for on-line preparation of mathematical documents", *Software concepts and Tools*, 1997, pages 80-89.

[4] Jarret, R and Su, P, "Building Tablet PC Applications", *Microsoft Press*, 2003.